



Galileo's Ship feat. Einstein's (Special) Principle of Relativity

El barco de Galileo y el principio de relatividad (especial) de Einstein

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INFORMACIÓN DEL ARTÍCULO	ABSTRACT/RESUMEN
Recibido el: 8/04/2026 Aceptado el: 16/09/2026 Keywords: Semantics, thought experiment, (special) principle of relativity, coordinate system, frame of reference Palabras clave: Semántica, experimento mental, principio de relatividad (especial), sistema de coordenadas, marco de referencia	Abstract: In this paper we identify Galileo's Ship as an Experience TE (TEEX), logically analyze it with help of methodology of TE Matrix, and discuss how its interpretations could contribute to logical formalization of Einstein's special principle of relativity. On surface analysis it may appear a series of six sub-TE, conducted on a ship, that one may recognize from personal experience of water travel. On logical deep analyses it is understood as instantiation of principle of relativity—with help of propositional logic it may be analyzed inductively, deductively, modally, and finally as material implication from equivalent frames of reference to laws of physics, and vice versa, which may render it a biconditional. We review some literature on Einstein's special principle of relativity, which may show overcomplex (e.g. nonclassical), physical and empirical mathematical interpretations, that, just like Einstein's $E=mc^2$, may provide for insightful considerations and imaginative visualizations of empirical spacetime situation, but may fall short as adequate and relevant (meta)logical formalization as, e.g., involving a crucial and indispensable theoretical relation between internal (e.g. Fin) and external frame of reference (e.g. Fex), that may define Galileo's Ship as well as Einstein's Elevator and Chasing a Beam of Light. Resumen: En este artículo identificamos el “barco de Galileo” como una experiencia TE (TEEX), la analizamos lógicamente con ayuda de la metodología de la

	matriz TE y discutimos cómo sus interpretaciones pueden contribuir a la formalización lógica del principio de relatividad especial de Einstein. A simple vista, puede parecer una serie de seis sub TE llevadas a cabo en un barco, que uno puede reconocer a partir de una experiencia personal en viajes por agua. En un análisis lógico profundo se entiende como una instanciación del principio de relatividad: con ayuda de la lógica proposicional puede analizarse de forma inductiva, deductiva y modal; y – finalmente– como implicación material entre marcos de referencia equivalentes y las leyes de la física, y viceversa, lo que podría convertirlo en un bicondicional. Repasamos parte de la bibliografía sobre el principio especial de la relatividad de Einstein, que puede presentar interpretaciones matemáticas excesivamente complejas (por ejemplo, no clásicas), físicas y empíricas, que al igual que la fórmula $E=mc^2$, pueden aportar reflexiones perspicaces y visualizaciones imaginativas de situaciones empíricas en el espacio-tiempo, pero pueden resultar insuficientes como formalización (meta)lógica adecuada y relevante, ya que –por ejemplo– implican una relación teórica crucial e indispensable entre el sistema de referencia interno (Fin) y el externo (Fex), que puede definir tanto el “barco de Galileo” como el “ascensor de Einstein” y la “persecución de un rayo de luz”.
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Introduction

Although the paper is firstly about *Galileo's Ship*, the second part, say, from section Principle of Relativity on, elaborates on Einstein's principle of relativity. Philosophically and logically speaking, the principle of relativity is about (logical) grammar (Wittgenstein 1922, 1953) of (principle(s) of) 'relativity' and 'equivalence,' and how to formalize it with help of available logical symbolisms. A major semantical problem is that there is no generally accepted agreement about senses and uses of these words, terms, concepts, and symbolisms (e.g. Copi et al., 2014, e.g. pp. 344-351; Sawa et al., 2013; Shieber, 1993 on equivalence). In proposals of logical formalization of this paper we will eventually apply an extended version of the propositional calculus including logical operators like biconditional $\Leftrightarrow$ and logical equivalence $\equiv$, which concepts and signs may be discussed as in accordance with Albert Einstein's (special) principle of relativity (please, see section Principle of Relativity and succeeding sections incl. Provisional Conclusion), analogous to logical formalization of Galileo's principle of relativity as introduced in *Galileo's Ship* (please, see preceding sections, such as surface and deep analyses, inductive, deductive, and modal formalizations).

Surface Analyses

The original text in translation of Stillman Drake may show quite clearly *Galileo's Ship* is about an experiment (E), not a thought experiment (TE), though it is more like a common tale of

experience recounted by Salviati than a scientific experiment, which may render it like an *Experience TE* (TE_{EI}) (instead of an *Experiment TE* (TE_{EX} – see Hertogh 2015, Ch. 3 Taxonomies, e.g. p. 130).

It sounds trustworthy to Sagredo who replies that although it did not occur to him to put these observations to the test when he was voyaging, he is sure that they would take place the way Salviati has described them.

In the 1967 English edition the *Galileo's Ship TE* is introduced in the margin of the text of the Second Day as a sort of crucial experiment like this “Experiment which alone shows the nullity of all those adduced against the motion of the earth” (Galilei, 1632/1967, p.186)

The TE reads like a manual of instructions to a (real) E, possibly performed by Salviati or Galileo and most probably resulting from common life experience with these situations. Galileo may have been in these situations many, many times—that is traveling by ship, on deck, in the hold, in the cabin--and through the years he may have noticed these phenomena as correspondences of effects between the situation when a ship is still and when it is in uniform motion, possibly having his account confirmed by experienced seafarers etc.

Although the passage does not live up to nowadays strict standards of a report of a scientific laboratory experiment, satisfying formal conditions and procedures, it surely is not an unperformable (T)E at least not in the sense that it is a TE that could not have been performed as a real E in those days for technical reasons or stage of progress of science (17th century, 1632), and it does not describe an 'essentially' imaginary situation that could not have been set in reality as well in those days. Instead, it seems a reflection from daily experience, possibly an informal experience wherein some features of daily life are observed in a scientific-like way and wherein some things have been put to the test e.g. jumping up and down, water dripping from a bottle into a bowl, smoke going up, swimming of fishes in a fishbowl, flying of (butter)flies in the air etc., observing how these things may differ between the situation of a stationary ship and a ship in motion, below decks and on deck.

The text has no TE indicators as 'Imagine ...', 'Consider...', but starts off with imperatives as “Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals....” and continues with a series of imperatives as Have a large bowl of water ... “... hang up a bottle ...”

(Galilei, 1632/1967, p. 186) Next, it has some type-1 conditionals predicting, telling causal chains, inferences like rules of physical law “(though *doubtless* when the ship is standing still everything *must* happen in this way)” (Galilei, 1632/1967, p. 187, italics added) trying to explain possible differences with a similar situation under slightly different circumstances, viz. doing the same things on deck “for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted.” (Galilei, 1632/1967, p. 187)

In fact, it is about six sub-TE, flying of little animals (TE₁), swimming of fish (TE₂), falling of drops into a vessel (TE₃), throwing something to a friend (TE₄), jumping with feet together up and down (TE₅), observing smoke from burning incense (TE₆).

These are six corresponding effects which are brought about by one cause, which is revealed in the end as by conclusion or explanation to prescribed or described observed or presumed observable phenome “The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also.” (Galilei, 1632/1967, p. 187)

Deep Analyses—Deduction after Induction

On deep analyses *Galileo's Ship* demonstrates to be both an inductive example and a deductive inference of *principle of relativity* holding that the equations describing the laws of physics have the same form in all admissible frames of reference.

That is, in all, say, enclosed spaces, inertial frames of reference water will drop down vertically, smoke will go up straight, etc.

Propositional logic

Suppose

[1]

$p \rightarrow q$ (P) (common) activities, effects as (P : set of common activities, effects) e.g., jumping up and down, water dripping down in bowl, smoke going straight up, regularly swimming of fishes, regularly flying of (butter)flies etc.

S₁ Setting 1--(ship is) at rest

S₂ Setting 2--(ship is) in uniform motion (cabin below decks),

Then,

$$S_1: p \rightarrow q$$

$$S_2: p \rightarrow q$$

$S_1 =_{\text{effects}} S_2$ is equivalent to w.r.t. *correspondences of effects between S_1 and S_2*

Or

$$S_1 \rightarrow P \quad [2]$$

$$S_2 \rightarrow P$$

$$S_1 =_{\text{effects}} S_2$$

Inductive Alternate—Formalization

$$S_1 \rightarrow P \quad [3]$$

$$S_2 \rightarrow P$$

$$(S_1 \vee S_2) \rightarrow P$$

that is S_1 and S_2 are replaceable or equivalent with regard to their effect on P, in Galilean words— correspondences of effects between S_1 and S_2 applied to specific situation of Galileo's Ship TE—common activities and effects P are inferred by S_1 and S_2 :

$$S_1 \rightarrow P \quad [4]$$

$$S_2 \rightarrow P$$

$$(S_1 \rightarrow P) \wedge (S_2 \rightarrow P)$$

$$S_1 =_{\text{effects}} S_2$$

that is S_1 and S_2 are replaceable or equivalent as regards their effect on (any relevant) Q

$$S_1 \rightarrow Q \quad [5]$$

$$S_2 \rightarrow Q$$

Deductive Alternate – Formalization

$$(S_1 \vee S_2) \rightarrow P \quad [6]$$

$$S_1 \rightarrow P$$

$$S_2 \rightarrow P$$

$$(S_1 \vee S_2) \rightarrow P \quad \text{i.e., } S_1 \rightarrow P, S_2 \rightarrow P$$

that is, if S_1 and S_2 are both admissible frames of reference, both S_1 and S_2 individually will yield P.

Modal Analyses

Though not an essentially modal TE, a modal logical analysis involving frame semantics and possible worlds (e.g. Kripke 1963), e.g., $\langle W, \models, R \rangle$, descriptive (K[ripke-style]) model involving set of (in)finite (P)W (possible worlds), accessibility Relation, satisfaction relation $\models$, would run.

Suppose [7]

$$\forall w \models p \rightarrow q$$

w_r at rest (with the ship standing still)

$\forall$ universal quantifier

w_m uniform motion (have the ship proceed with any speed you like, as long as the motion is uniform),

Then,

$$Aw_r \models p \rightarrow q$$

$$Aw_m \models p \rightarrow q$$

$$w_r \models_{\text{effects}} w_m$$

A modal logical analysis is not necessary as it is about possibly actual worlds in (the actual world of) Planet Earth, i.e., it is about easily realizable situations as in a real experiment. It is physically possible to (re)build a ship in a laboratory like in movie scenes, or to (re)perform Salviati's/Galileo's instructions in an experimental setting of a real ship.

Because the TE can be performed as a real E, its premises are not essentially modal, and they relate immediately to the mesolevel of Planet Earth. Non-modal analyses are sufficient and strong enough; modal analyses are not required in this case.

Principle of Relativity

The principle of relativity (in restricted sense) can be formalized as material implication from disjunction of frames of reference to laws of physics, and vice versa, which may render it a biconditional, like this:

Suppose [8]

L (equations of) Laws of physics (as expressed in quantities) $F_1, F_2, F_3 \dots F_n$
admissible (inertial) Frames of reference 1, 2, 3 ... n

$\vee$ disjunction

$\leftrightarrow$ biconditional

As it is similar to *Galileo's Ship* TE

$F_1 \leftrightarrow L$ [9]

$F_2 \leftrightarrow L$

$(F_1 \vee F_2) \leftrightarrow L$

Generalizing

$(F_1 \vee F_2 \vee F_3 \dots F_n) \leftrightarrow L$ [10]

Einstein's (Special) Principle of Relativity

Mathematical Formalization of Mass Energy Equivalence

Although this paper is about a logical formalization we will first introduce the world famous mathematical formalization, which is about mass energy equivalence-- $E = mc^2$, energy (E) equals (=) mass (m) times square (^{2}) of speed of light in vacuum (c – ab. 300.000 km/s), that is derived from Einstein (1905/1923/1952b), 'Does the Inertia of a Body Depend upon its Energy-Content', (and is only about one equivalence relation viz. mass – energy), which appears like $m = L/c^2$ in this article, where L stands for energy, m for mass and c for speed of light in vacuum

If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2 .

The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that the mass of a body is a measure of its energy-content; if the energy changes by L , the mass changes in the same sense by $L/9 \times 10^{20}$, the energy being measured in ergs [erg is a unit of energy equal to 10^{-7} joules, 100 nJ], and the mass in grammes.

....

If the theory corresponds to the facts, radiation conveys inertia between the emitting and absorbing bodies. (Einstein, 1905/1923/1952b, p. 71)

Logics of Quantum Mechanics

Quite a lot of literature that may be found by e.g. Google *Scholar* by keyword logical formalization of principle of relativity (Andréka et al. 2006, Gömöri and Szabó 2015, Madarász et al. 2017, Székely 2009, Molnar and Székely 2015), is more about above empirical reality of mathematical physical formula and microphysics than about Einstein's principle of relativity that literally holds for *all* laws of physics, not only or particularly for e.g. quantum mechanics, on which Einstein never really agreed (opting instead for local realism).

We will discuss aforementioned literature and recent updates in philosophical literature, debate (Carnap)/Quine/Putnam vs. Kripke on quantum logic, although they are more about development of alternate logic for microphysics than about Einstein's first postulate of relativity.

Andréka et al. (2006), Gömöri & Szabó (2015), Madarász et al. (2017), Székely (2009), Molnar & Székely (2015) are related and referring to each other, e.g., Hajnal Andréka and Judit Madarász were thesis advisors to Gergely Székely (2009) PhD thesis.

Madarász et al. (2017) mentions that Einstein derived principle of relativity from *Galileo's Ship*, but, next, Madarász et al. proceed in attempts to find a first-order logical interpretation (in threefold, $\text{SPR}_{\mathcal{M}}$, SPR^+ and $\text{SPR}_{B,IOb}$), which turns out to be an axiomatized, physically based formalization, that goes far beyond vocabulary, requirements and limits of first-order logic, that is restricted to some logical operators (e.g. negation, disjunction, conjunction), variables, constants and parentheses--Madarász et al. (2017) introducing e.g.

.... We will use the first-order logical (FOL) 3-sorted language

$$\mathcal{L}_0 = \{IOb, B, Q, 0, 1, +, \cdot, W\}$$

as a core language for kinematics. In this language

- *IOb* is the sort of *inertial observers* (for labelling coordinate systems);
- *B* is the sort of *bodies*, i.e. things that move;
- *Q* is the sort of *quantities*, i.e. numbers, with constants 0 and 1, *addition* (+) and *multiplication* ($\cdot$);
- *W* is the worldview relation, a 6-ary relation of type $IOb \times B \times Q^4$.

The statement $W(k, b, p)$ represents the idea that 'inertial observer *k*

coordinatises body b to be at spacetime location p .' (p. 534)

which is overcomplex for the task at hand, logical formalization of principle of relativity, but may indicate how the physical situation could be pictured as involving spacetime.

Some more formalizations that are proposed in the literature (Andréka et al., 2006, Gömöri and Szabó 2015, Székely 2009, Molnar and Székely 2015) may seem overcomplex, too, as they focus on quantum physics while principle of relativity holds for *all* laws of physics (including Newtonian mechanics—see Norton's account below), and they are rather empirical and physical than logical. The complexity of quantum mechanics, possibly involving photons, Maxwell's equations, Einstein's field equations, Minkowski's spacetime, has been confused with the (special) principle of relativity itself, and, e.g., epistemic principle of simplicity would recommend a simpler and more basic formalization.

In Section 3.1, 'Special relativity in four simple axioms,' of *First-Order Logic Investigation of Relativity Theory with an Emphasis on Accelerated Observers* (2009) Székely introduces 'the following axiom system' (Székely, 2009, p. 20):

SpecRel:= { AxSelf0, AxPh, AxEv, AxSymDist } [11]

First axiom, AxSelf0, may hold for observers, second axiom, AxPh, for speeds of photons, third axiom, AxEv, holds 'events observed by inertial observers are the same', fourth axiom, AxSymDist, involves symmetric distance axiom; the axioms include first-order logical interpretation involving inertial observers (m, k), set of observers ($Ob\ x$) etc. (see Madarász et al. 2017).

Székely investigates into special relativity theory, and his first order (predicate) logical (FOL) definitions of axioms, which show some similarity to Einstein's postulates, do not hold for *all* laws of nature, as stated by Einstein's first postulate, but particularly for relativity theory.

Review of logical literature on (e.g.) quantum logic, may show update of discussion of e.g. Hilary Putnam (1968/1979) attempts of an empirical, quantum logic (partly based on e.g. Birkhoff and von Neumann, 1936), and WVO Quine's behaviorist meta philosophical gradualism (e.g. Quine 1951, 1960, 1969), or anti-exceptionalism about logic (updated by e.g. Williamson 2013), in Saul Kripke's posthumously published 'The Question of Logic' (2023),

derived from lectures and seminars in 1974, 2014, 2015, 2020.

As we have already remarked (Hertogh, 2025b) on the *Liar* paradox, Kripke's use of Kleene's three-valued logic in 'Outline of a Theory of Truth' (Kripke, 1975), should—as according to Kripke himself— not be mistaken for a proposal as 'of those who favor abandoning standard logic "for quantum mechanics"' (Kripke, 1975, p. 700, n. 18), and in the same place Kripke emphasizes that he would like to hold on to classical bivalent logic (see Hertogh, 2025b, pp. 103-104), so, it won't be surprising that Kripke does not accept Putnam's non-classical (non-distributive) proposal of logic of quantum mechanics (Putnam, 1968/1979), nor e.g. Quine related 'adoption' view of logic ('choose to "adopt" one and later reject it in favor of another', 'to adopt x logic', x standing for one's favorite formal system – Kripke, 2023, pp. 1-2), against which issue there are given more arguments in Kripke (like 'But the adoption of intuitionistic logic as opposed to "classical" logic amounts to systematically forswearing certain classically valid inference (Putnam, 1962, p. 51, quoted in Kripke, 2023, p. 2). Countering Carnap (1934/1937) and Quine (1951) projects of empirical logics, Kripke (2023, e.g., p. 16) argues that one may modify a particular hypothesis, like 'All raven are black',:

The logical positivists – say Carnap – thought we could adopt one logic or another, one language or another, and that a principle of tolerance should be exercised. One can adopt any kind of laws for the logical connectives one pleases ('... in logic, there are no morals', Carnap 1937, pp. 51-2). ... Only some kind of scientific utility should make you prefer one to another. They thought that it was just a convention which logic we 'adopt' (whatever that may mean). But one is completely free to choose. (Kripke, 2023, p. 9 – referring to Carnap, 1937, pp. 51-52, and in a note to p. xv on 'arbitrarily' choosing 'any postulates or any rules of inference').

in a note adding that this is a 'familiar oversimplification of Carnap's and the positivists' views.' Against this reservation, Quine made the 'revolutionary suggestion' that "'adopting" a logical law was no different from adopting a scientific hypothesis'

.... Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle? (Quine, 1951, p. 40, quoted in Kripke, 2023, pp.10-11)

Arguing that classical logic is older, more traditional and established than any new development of science (e.g., Kripke, 2023, p. 13, e.g. 'A maxim of conceptual conservatism, or of "minimum mutilation," should lead us to generally prefer the alternative that does not change logic.'), next that '[t]here could be no neutral ground in which to discuss the drawing of consequences independently of logic itself' (19), and, finally, that there are not different logics, but (classical) logic and 'intuitive reasoning' (pp. 32, 33).

Countering Putnam's omission of the distributive law, Kripke reestablishes classical propositional logic, showing it is consistent, while Putnam's alternate of 'quantum logic' is inconsistent and unsound (Kripke, 2023, p. 32).

Overall, even less so than in case of *Liar*-type sentences (see Hertogh 2025b), Kripke does not consider e.g., 'something more complex, such as Hilbert space' (Kripke, 2023, p. 25) a reason to reject or revise classical logic.

Kripke concludes "I haven't considered all alternatives but why not say this: there aren't different logics, there is only logic, that is only reasoning. There are different formal systems; we use logic to reason about them." (2023, p. 32)

Kripke *does* acknowledge different (e.g. non-Euclidean) geometries and formal systems; 'introducing'--instead of adopting--'a new logic' ('but only in a loose sense' e.g. new informal interpretations, sound and/or complete principles, new connectives, new laws); updating old formal systems by correcting fallacies, fallacious proofs; 'special situations' which may require a 'modified semantics' (e.g. Łukasiewicz's and Kleene's three-valued logics may handle vague or partially defined predicates and truth value gaps; they are 'not competing ideas, but alternative approaches') (Kripke, 2023, pp. 32-33).

Logical Formalization of Special Principle of Relativity

The principle of relativity as derived from e.g., *Galileo's Ship* is considered Einstein's first postulate of relativity theory, and phrased like this (still in terminology of Galileo's 'system of coordinates' etc.),

If a system of coordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form, the same laws hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K . This postulate we call the 'special principle of relativity.' (Einstein, 1916/1923/1952c, §1, 111)

With regard to phrasing of (special) principle of relativity by Galileo and Einstein question remains what exactly is 'hold good in relation to' or 'equivalent'/'equivalence' (of K' and K) as this relation is usually referred to in secondary literature as well as by Einstein himself (e.g. Einstein, 1916/1923/1952c, p. 111 – ... The word 'special' is meant to intimate that the principle is restricted to the case when K' has a motion of uniform translation relatively to K , but that the equivalence of K' and K does not extend to the case of non-uniform motion of K' relatively to K)--*what* (kind of) equivalence (to *what*), holding good in *what* (kind of) relation to (*what*), and *how*? Hans Reichenbach in *The Philosophy of Space-Time* (1958 trans.) opts already in 1920s it is not about 'coordinate systems' only, but coordinate systems with their gravitational fields,

.... The coordinate systems themselves are not equivalent, but every coordinate system with *its* corresponding gravitational field is equivalent to every other coordinate system together with *its* corresponding gravitational field. Each of these covariant descriptions is an admiss[i]ble representation of the *invariant* state of the world. (Reichenbach, 1928/1958, p. 237, quoted in Salmon, 1977, p. 31)

Samokhvalov in 'Logic and the Relativity Principle' (2000) wants to emphasize it is not about the 'system of coordinates' or laws, but their equations

.... In other words, the *equations* of physics have *the same form* in both frames α and β (or, as the saying goes, are *invariant* under the transition from frame α to frame β), but the *results of concrete measurements of concrete* events must, generally speaking, *change* under this transition. (Samokhvalov, 2000, p. 233)

Samokhvalov, replacing Galileo's coordinate system by axiomatic system, redefines 'generalized relativity principle' as 'Well, the logical content of special relativity is nothing else than a metasentence about two concrete (considered a moment ago) axiomatic systems T and W ' (Samokhvalov, 2000, p. 234). John Norton is a preeminent Einstein scholar, who contributed to pivotal Horowitz & Massey (1991) TE collection on Einstein's TE (Norton 1991), who debates with e.g. James Brown on nature of physical laws, defending empiricism (Norton, 1996, 2004a, b), and may provide for bibliographical information on e.g. Einstein's *Chasing a Beam of Light* (Norton, 2005, 2011 - see Hertogh 2026 forthcoming), as well as for current topic of (special) principle of relativity (e.g. Norton, 1989, 1993, 2024). Following Norton's account, that features frames of reference and quantities (instead of equations – see

Madarász et al. 2017 above) and *all* laws of physics (instead of only or particularly quantum mechanics, as in Madarász et al., (2017), Szeleky (2009), or gravitation (as in Reichenbach's quote above) –e.g., electromagnetism (see e.g. first pages of Einstein, 1905)

The first principle, the principle of relativity, says something about all the laws of physics. To state the principle, we must first express those laws in a particular way, that is, in terms of the quantities defined within an inertial frame of reference.

Take Newton's first law of motion, for example. (It is a law that also holds in special relativity.) It says in Newton's original wording:

Every body continues in its state of rest, or in uniform motion in a right [straight] line, unless it is compelled to change that state by forces impressed upon it.

The terms used in this formulation of Newton's first law require that we refer to some inertial frame of reference. "State of rest" means that the body remains at the same position in the inertial frame over time. "Uniform motion" means that the body covers the same distance in the same time, where distance is measured by the frame's measuring rods and time is measured by the frame's clock.

The principle of relativity asserts:

The laws of physics are the same in all inertial frames of reference.

That just means this. If we express some law of physics using the quantities of one inertial frame of reference, the resulting statement of the law will be exactly the same in any other inertial frame of reference. (Norton, 2024, p. 8-9, bold replaced by underline)

As is evident from Norton's example, as Einstein adds himself to his 1916 formulation of special principle of relativity (Einstein, 1916/1923/1952c, p. 111 – 'The special theory of relativity is based on the following postulate, which is also satisfied by the mechanics of Galileo and Newton.'), and as we may know from high school, Newton's first law of motion holds for and in all inertial situations.

Logical formalization of principle of relativity may constitute another 'special situation' (Kripke, 2023, p. 33), but instead of doubting what empirical or quantum reality, coordinate systems, laws of physics or logic may entail, we could--following Kripke--hold on to classical, bivalent logic and basic principles of logic (like identity, excluded middle, noncontradiction), and see that the relevant *logical* issue is here about equivalence, 'are the

same in' (Norton, 2024) or '[the same laws] hold good in relation to [any other system of coordinates K']' (Einstein, 1916/1923/1952c, §1, p. 111) as in our provisional formalization after *Galileo's Ship* in third section, that features Frames of reference (F) and Laws of physics (L)--as by material implication, let's say causal graph, from laws of physics to equivalence of frames of reference, since the physical laws are affecting the frames of reference similarly or equivalently, and vice versa, which may render it a possibly (causal) biconditional, or possible bidirectional causation on (meta)physical interpretation (causation is a philosophically speaking cumbersome concept and possibly rather metaphysics than physics, but this issue is beyond the scope of current research).

$$(F_1 \vee F_2) \leftrightarrow L \quad [9]$$

As after [2], [3], [4], that is to say, F_1 and F_2 are mutually replaceable or logically equivalent, in fact ($F_1 \equiv F_2$) instead of only commutatively replaceable as ($F_1 \vee F_2$), with regard to the effects on and from L, so we could add subscript 'effects' again, $\leftrightarrow_{\text{effects}}$, and use a double arrow to indicate it is not just about material implication from L to Fs, but also, vice versa, from Fs to L, $\Leftrightarrow$ (both $\rightarrow$ or $\Rightarrow$ and $\leftarrow$ or $\Leftarrow$), or $\Leftrightarrow_{\text{effects}}$, retaining the double arrow--instead of static tribar ($\equiv$) used for the first logical operation of the formula ($F_1 \equiv F_2$)--since the second logical operation of the formula is like a biconditional, double material implication, in physical terms possibly double or mutual causation

$$(F_1 \equiv F_2) \Leftrightarrow_{\text{effects}} L \quad [12]$$

We could possibly consider (structural) isomorphism, but we have seen in Norton 2024 that it is about sameness of application to different situations, not about a structural resemblance between the laws and the situation to which the laws are applied, so, although it could be about metalogic, isomorphism may be hopeless metaphysical, assuming external reality is like our formulas, like medieval conceptions that the external world originates from books of creation etc. Additional argument to reject this option is that the sign for isomorphism, structural identity ($\cong$ Unicode U+2245, which sign is also used for 'approximately equal to') comes from mathematics and science and not from logic, it is not a logical operator.

One may argue that subscripts to logical operators are not allowed in propositional

calculus, but we see use of sub and superscript more and more in all kinds of logical, scientific, mathematical publications, and could just adjust to these seeming unwritten habits and rules, or acknowledge that our formalization transcends classical proposition logic, classical, standard logic etc. and needs clarification by subscripts, in fact indicating *aspect, dimension, perspective* or *context* of equivalence, sameness at hand, compare Wittgenstein *seeing under an aspect* (Wittgenstein, 1953, : Pt. II, section xi), Gendler's 1996 constructivism (Gendler, 1996, p. 198) and Daoist aspect thinking (e.g. Lǎozǐ ca. 571–471, BCE). If we want to remain within classical proposition logic we may simplify to:

$$(F_1 \equiv F_2) \Leftrightarrow L \quad [13]$$

which may be fully justified if we move the *aspect, dimension, perspective, context*, etc. issue to the introducing definitions of logical operators and proposition letters e.g.

Suppose

$\Leftrightarrow$ biconditional, double material implication, which logical sense may cover *transference* of physical effects.

The physical laws affect the frames of reference, $L \rightarrow F$ (invariance), and the frames of reference affect the physical laws, $F \rightarrow L$ (inertia, pseudo-forces), too. First, it matters if the frame of reference is inertial or noninertial, and we may already have confined definition of frames of reference to admissible or *inertial* frames in the introductory definitions of logical operators, relations, and proposition letters. Second, the frames of reference may affect how the application of the physical laws may turn out, e.g., in case of Galilean gravity of falling objects, the falling speed is mediated by, e.g., quality of its matter, e.g., solid, liquid or gaseous, which secondary parameters like density, viscosity, friction, (air, drag, fluid, water etc.) resistance, shape etc. may slow down the gravitational force (see Galilei, 1638/1914, Hertogh 2023). Third, it may matter if it is about mesolevel of Planet Earth, where Galilean-Newtonian classical mechanics apply (by approximation), and micro and macrolevel of Planet Earth, submolecular and cosmological level, where modern physics of e.g. relativity theory and quantum mechanics apply.

We may escape from relativism of contextualism by explaining it is not relativism but exactly *relativity* as after name of the theory at hand, possibly specifying aforementioned

differences in application of laws of physics as between mesolevel or micro and macrolevel of Planet Earth.

With regard to quantum mechanics, there is an additional problem of application of laws of physics to external reality as discussed in e.g. EPR (Einstein, Podolsky and Rosen, 1935). The conception of non- Euclidean physical spacetime reality is complicated by different interpretations, such as the Copenhagen view, which remains skeptical about underlying reality, the quantum mechanical view, e.g. wave-particle duality, e.g. Niels Bohr, and the (local) realism view, e.g., Einstein, who in EPR argues the quantum mechanical description of physical reality, given by wave functions, cannot be complete, e.g., because, if two quantities, say, P and Q , cannot be simultaneously predicted, 'they are not simultaneously real' (Einstein et al., 1935, p. 780), in 1949 adding e.g. 'I am, in fact, firmly convinced that the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this [theory] operates with an incomplete description of physical systems' (Schilpp, 1969, p. 666, from 'Reply to Criticisms').

However, these are again digressions into empirical physical reality, which are not foremost relevant for a logical formalization of principle or relativity, which is the main aim of this paper. The former may confuse and complicate the latter as we have seen in the last preceding sections.

Thought Experimental View

As usual in TE research, our interpretation is based on close surface reading of the TE text, in this case mediated by Norton's scholarly work on Einstein. Our formula as derived from the text translates the literal text and may leave some questions open when there are some ambiguities in the TE text. Also, we only interpret the text verbally and symbolically without judging truth, soundness etc. of theory at hand, however with some restrictions as of progress of science and society, involving global cross- culturalism and environmental pragmatism, the first condition saying it should hold in more than one culture, which is obvious here since we witness application of theory of relativity all over the world, as say from US to China, and the second condition saying it should not be resulting in harm to human health and ecology of Planet Earth, which could indeed be a problem here as regarding some applications of these modern science theories as not only in warfare

(nuclear weapons – see *Russell- Einstein Manifesto* 1955, criticizing A-bomb on Nagasaki and Hiroshima), but also more possibly damaging societal technological applications, such as nuclear power plants with non-biodegradable nuclear waste, to mention only the most well-known examples comparable to harm from applications of physicochemical microstructure theories (such as water = H₂O – see e.g. Kripke, 1980, 2019).

Although the empirical physical situation may be complex, referring to non-Euclidean mathematics and spacetime, the logical interpretation of Einstein's special principle of relativity may remain within (metalogical) classical proposition logic to some extent, using only two logical operators or relations, equivalence (replacing previous option of disjunction) and biconditional, next generalized by sequence notation, which is however not strictly necessary, as usually the setting may remain within a limited number of 'systems of coordinates' or 'frames of reference', as in the prototypical examples of *Galileo's Ship* and Einstein's TE, such as *Elevator* (see Hertogh 2024a), *Chasing a Beam of Light* (Einstein & Infeld, 1938, 226–235, Hertogh, 2024a), *Chasing a Beam of Light* (Einstein, 1949, 49–50, 52–53/Einstein, 1969, 52–53, 54–57, Einstein, 1956, 10, Wertheimer, 1959, 214–215, Hertogh 2026 forthcoming), which have no more than two observers, an *internal* or *inside observer* and *external* or *outside observer*, representing two different frames of reference with in some senses opposing or contradictory effects. As already indicated in paper about Zhuangzi (Hertogh 2024b : e.g. 91-94), it is well-possible that more, if not all, TE are reducibly understood as ambiguity between two *points of views, perspectives, aspects* etc. as e.g. in example of Sorensen's immigration of the supposition operator (SBp → BSp – Sorensen, 1992, pp. 202-203), or change from inquit of indirect speech to subject of direct speech patterns, like we may show migration of dream operator in e.g. Martin Luther King Jr.'s *I Have a Dream*, Zhuangzi's *Butterfly Dream*, Einstein's *Chasing a Beam of Light* (resp. Hertogh, 2022, 2024b, 2026 forthcoming – which double structure of TE, generally speaking, could be comprehended from intentional ambiguity of phenomenological act of imagining, e.g. Sartre 1940/1966), like disentangling of zero and metalevel in *Liar* paradoxes (Hertogh, 2025b), returning to just before the generalization of [10], replacing $\vee$ and $\leftrightarrow$ by the more accurate $\equiv$ and $\Leftrightarrow$,

$$(F_{in} \equiv F_{ex}) \Leftrightarrow L \quad [14]$$

This answers Reichenbach's question that the coordinate systems don't include

gravitational fields (but the laws include gravitation) and Samokhvalov's question that it is not only or particularly about the equations in which laws of physics are expressed but about the effects of the laws of physics on coordinate systems, although one may assume that the laws of physics are expressed in equations as derived from maths, and we may agree on Samokhvalov's conclusion that principle of relativity is—in our terms—a metalogical statement about two frames of reference.

Although we sometimes venture into more complex, e.g., nonclassical, resolutions, like considerations on quantum logic in this case, it appears quite often (metalogical) classical enthymemes may do as analyses of thought experiments.

Application of mathematics may result in mismatch between laws of physics and phenomenology, as that e.g. Galileo's laws and discoveries may still appear counter-intuitive since it may still *seem* that the sun is rising and setting, and that falling speed of any object may depend on weight instead of gravitation constant, friction, air resistance etc.

Kripke is repeating more than once that—in terms of Richard Rorty 1979—our 'final vocabulary'—is classical logic, that our final vocabulary is classical logic, that we decide eventually by 'intuitive reasoning', etc., thereby confirming convictions about the use of logic similar to logicians like Józef Bocheński and Alfred Tarski, e.g.

The point is that logic, even if one tries to throw intuitions to the wind, cannot be just like geometry because one cannot adopt the logical laws as hypotheses and draw the consequences. You need logic in order to draw these consequences. (Kripke, 2023, p. 19)

Philosopher-theologian Bocheński on the act of faith in *The Logic of Religion* (1965),

.... This had to be said for two purposes. On the one hand, it must be shown that it is not the business of logic as such to interfere with the work of these other sciences, in particular that of theologies and Buddhologies, *except insofar as it may examine the correctness of their reasoning*. For logic as such cannot assume any p-sentences and cannot, therefore, be competent in any such field. On the other hand, the statement may be useful to clarify a point which does not always seem to have been understood: namely, that every attempt to substitute psychological, phenomenological, or theological considerations for logical analysis is illegitimate here. *When justification*

is concerned, it must be repeated, logic alone is competent, and no other discipline.

(Bocheński, 1965, pp. 134–135, italics added)

Alfred Tarski in a section on the importance of logic for other sciences in his logic textbook *Introduction to Logic and to the Methodology of Deductive Sciences* (1936/1994)

.... Let us say this in a different way: logic is justly considered the basis ... of all other sciences, even if only for the reason that in every argument we employ concepts taken from the field of logic, and that every correct inference proceeds in accordance with its laws. ... (p. 101)

These convictions may add justification to our search for a logical formula of the principle of relativity, that it is scientifically and logically relevant as in formalizations of Galileo's and Einstein's TE and theories, and that it may be expressed in a vocabulary as elementary as classical truth-functional propositional logic.

Provisional conclusion

The contextual special meaning of biconditional featuring effects of laws of physics on different frames of reference could be explained in the preceding introducing logical definitions of terminology, so that the formula itself may remain within classical propositional calculus.

Finally, we could generalize our formalization as an axiom using metavariables of a potentially infinite series of equivalent frames of reference ($\varphi_1, \varphi_2, \varphi_3 \dots \varphi_n$) and relevant set of laws of physics (λ)

Suppose [15]

λ (relevant set of) laws of physics

— biconditional, bidirectional material implication

$\varphi_1, \varphi_2, \varphi_3 \dots \varphi_n$ (inertial) frame of reference 1, 2, 3 ...
n

$\equiv$ equivalence as w.r.t. above biconditional

$\lambda \Leftrightarrow (\varphi_1 \equiv \varphi_2 \equiv \varphi_3 \dots \varphi_n)$

W.r.t. (*with regard/respect to*) is a common abbreviation in maths and science, indicating *differentiation* or *specification* of a relation relative to a specific *variable* or *aspect*, *dimension*,

perspective, context, application etc. of the relation. We haven't found a logical sign for it yet, but it could be indicated by a subscript as in $\Leftrightarrow_{\text{effects}}$ (see [1], [2], [4], [7], [12]), which, however, is not preferred because it isn't part of propositional calculus and 'effects' refers to physical interpretation.

Physical interpretation may read--In formula $\lambda \Leftrightarrow (\varphi_1 \equiv \varphi_2 \equiv \varphi_3 \dots \varphi_n)$, $\equiv$ signifies *correspondence* of physical effects among frames of reference, $\varphi_1, \varphi_2, \varphi_3 \dots \varphi_n$; and $\Leftrightarrow$ signifies *transference* of physical effects between frames of reference, $\varphi_1, \varphi_2, \varphi_3 \dots \varphi_n$, and physical laws λ , and vice versa; $\varphi_1 \equiv \varphi_2 \equiv \varphi_3 \dots \varphi_n$ in this formula wants to say $\varphi_1, \varphi_2, \varphi_3 \dots \varphi_n$ are *equivalent* to each other *w.r.t. transference* of physical effects, and so they may show *correspondences* of effects.

Special principle of relativity in words of John Norton (2024), the laws of physics are the same in all inertial frames of reference.

Extended argument and Logic TE theory does not require axiomatization on hypothesis of *descriptive* (instead of prescriptive) *semantics* – As the possible worlds of TE arguments, e.g. premises and conclusions, pick out accessible possible worlds, as from axioms to principles (as possibly with help of search engines like *Google, Google Scholar* and *Baidu*), unto the argument is both formally and informally logically validated (e.g. premises and conclusions are true, logical inference is valid, argument is sound), we don't need to prescribe any forcing formal validation function.

Range of societal applications of physicochemical microstructure theories is restricted by 21st-century conditions of *global cross-culturalism* and *environmental pragmatism*, as to see to it that theories are not confined to only one culture, and to prevent environmental harm to human health and ecology of Planet Earth.

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