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BERTRAND RUSSELL AND LUDWIG WITTGENSTEIN, PERSONAL AND ACADEMIC RELATIONSHIP

BERTRAND RUSSELL Y LUDWIG WITTGENSTEIN, RELACIÓN PERSONAL Y ACADÉMICA

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INFORMACIÓN DEL ARTÍCULO	ABSTRACT/RESUMEN
	Abstract: In this article, the author analyses the relationship between two prominent philosophers of the 20th century in Europe and Great Britain—Ludwig Wittgenstein and Bertrand Russell. According to a lot of correspondence available nowadays we can reconstruct not only the environment of thought in Cambridge in the beginning and the first half of the 20th century but to find out some very personal, subjective grounds for the changes of relationship between thinkers, misunderstandings between them. Such a kind of biographical-historical reconstruction does not interfere but helps to understand better the origin, development, and criticism of philosophical ideas and theories. In this context, the personal relationship between Russell and Wittgenstein, their friendship for more than 30 years, and mutual assistance, both intellectual and business, played a big role in the development and formation of both philosophers as philosophers. Even mutual disagreements and criticism of ideas over time helped them to see the shortcomings and sometimes even complete failure of their theses and statements. First, the author describes in detail the role of Russell in Wittgenstein's life: first meeting and inspiration to do philosophy, support in studies, assistance in the publication of Wittgenstein's first book, support, and facilitation in returning to Cambridge in 1929, obtaining PhD degree and receiving Trinity College grant. Second, the author considers the fundamental points of philosophical disagreements between the two philosophers and Wittgenstein's critique of Russell's ideas. **Resumen:** En este artículo, el autor analiza la relación entre dos destacados filósofos del siglo XX en Europa y Gran Bretaña: Ludwig Wittgenstein y Bertrand Russell. De acuerdo con una gran cantidad de correspondencia disponible hoy en día,

podemos reconstruir no solo el entorno de pensamiento en Cambridge a principios y la primera mitad del siglo XX, sino también encontrar motivos subjetivos y personales para los cambios de relación entre pensadores, malentendidos entre a ellos. Tal tipo de reconstrucción biográfico-histórica no interfiere, sino que ayuda a comprender el origen, desarrollo y crítica de las ideas y teorías filosóficas. En este contexto, la relación personal entre Russell y Wittgenstein, su amistad durante más de 30 años y la ayuda mutua, tanto intelectual como empresarial, jugaron un papel importante en el desarrollo y formación de ambos filósofos como filósofos. Incluso los desacuerdos mutuos y la crítica de ideas a lo largo del tiempo les ayudaron a ver las deficiencias y, a veces, incluso el fracaso total de sus tesis y declaraciones. Primero, el autor describe en detalle el papel de Russell en la vida de Wittgenstein: primer encuentro e inspiración para hacer filosofía, apoyo en los estudios, asistencia en la publicación del primer libro de Wittgenstein, apoyo y facilitación para regresar a Cambridge en 1929, obtener el doctorado y recibiendo la beca del Trinity College. En segundo lugar, el autor considera los puntos básicos y fundamentales de los desacuerdos filosóficos entre dos filósofos y la crítica de Wittgenstein a las ideas de Russell.

Introduction

In 1908 young Ludwig Wittgenstein, on his father's advice, went to England to study engineering. Earlier in 1906 after finishing school and getting his matriculation, he decided to study engineering in Berlin at the *Technische Hochschule in Charlottenburg* after reading Franz Reuleaux's '*Theoretische Kinematik*' (1875). However, he was not satisfied with studying in Berlin, so he was looking for alternatives. In 1908 he enrolled at the College of Technology in Manchester. In the autumn of this year, he started his studying and practice at Manchester University as a research student in the Department of Engineering, working on the development of a 'motorless' aeroengine. While developing the propeller, Wittgenstein faced mathematical problems. And it piqued his interest more than the technical development of motors. Wittgenstein discussed mathematical problems with his colleagues in the engineering laboratory. Much later, one of Wittgenstein's pupils, remembering his teachings, suggested that Wittgenstein was seeking the truth all his life, "He sought the truth at first in engineering, then in mathematical logic". (Hijab, 2001)

According to von Wright (1955), Wittgenstein asked someone for advice about literature on the foundations of mathematics and was advised to read Bertrand Russell's *The Principles of Mathematics*, first published in 1903, after reading this book

Wittgenstein decided to write to Russell. Von Wright claims that Wittgenstein was influenced by Russell's study, and that let him apply to Frege's works. In von Wright's words, "It was probably it which led him to study the works of Frege. The 'new' logic, which in Frege and Russell had two of its most brilliant representatives, became the gateway through which Wittgenstein entered philosophy". (1955, p. 530) However, Russell was wrong, when he wrote that Wittgenstein did not know Frege before he came to Cambridge. (Russell, 1951) According to Hermine's notes, it was Frege who advised Wittgenstein to go to Cambridge and study with Russell. Wittgenstein himself told von Wright, that after he had decided to give up on studying engineering, he first went to Jena to meet Frege and discuss his subsequent plans with him (von Wright, 1955). Hermine, Ludwig's sister wrote in her memoir that her brother was working on a philosophical work, and he wanted to present it to Frege. It had happened in summer when Wittgenstein stopped in Jena on his way from Vienna to England. Frege endorsed Wittgenstein's philosophical thoughts and, also, advised him to continue his studies under Russell in Cambridge. In the preface to *Tractatus*, Wittgenstein (1922) wrote, "I will only mention that to the great works of Frege and the writings of my friend Bertrand Russell I owe in large measure the stimulation of my thoughts". (p. 23)

The years before WWI were the years of intellectual activity in Cambridge, especially in the field of logic and mathematics. G. E. Moore, one of the most influential logicians and philosophers in Cambridge at that time, published in 1899 *The Nature of Judgement*, in 1903 his famous *Principia Ethica*, the same year *Review of Franz Brentano's 'The Origin of the Knowledge of Right and Wrong'*, and *Refutation of Idealism*, in (1905-6) *The Nature and Reality of the Objects of Perception*. In 1903, G. Frege published the second volume of *Grundgesetze der Arithmetik*. In 1910, B. Russell published the first volume of *Principia Mathematica* written in collaboration with A. N. Whitehead, (Whitehead & Russell, 1910). This was the intellectual atmosphere in Cambridge in 1910 when Wittgenstein arrived. Wittgenstein took an interest in new logic represented by Frege and Russell. Wittgenstein had a historical-philosophical background. Earlier he had read Schopenhauer, Boltzmann, and philosophical aspects of Reuleaux's *Kinematics of Machinery*.

In 1912 Wittgenstein still had doubts whether to continue his studies in aeronautics and engineering or to switch completely to philosophy. Russell advised him to write an essay on any subject. Russell was impressed by the very first sentence and asked him to devote himself to philosophy. Russell wrote in a letter from October 18, 1911, to Ottoline Morell about Wittgenstein. "He turned out to be a man who had learnt engineering at Charlottenburg, but during his course had acquired, by himself, a passion for the philosophy of math's, and has now come to Cambridge on purpose to hear me" (Russell cited in Nedo, 2011). At the beginning of June 1912 Wittgenstein was admitted by the Degree Committee of the Special Board for Moral Science as Advanced Student to a course of research at Trinity College and "Mr. Bertrand Russell" was asked to "be kind enough to act as the Director and Supervisor of the Student". (Keynes in Wittgenstein, 1974, p. 1) Wittgenstein studied *Logic and the Foundations of Mathematics* with Russell and *Psychology* at Moore's lectures. His tutors were, first, the mathematician J. W. L. Glaisher, and later W. M. Fletcher. Wittgenstein developed friendships with Russell, Whitehead, Moore, J. M. Keynes, and mathematician G. H. Hardy. Moore recalled this story in a letter to F.A. Hayek. During his first stay in Cambridge (1911-1913) Wittgenstein became acquainted and established connections with the most famous and influential philosophers of that time, i.e., B. Russell, G. Moore, and J. M. Keynes, who were active members of the *Apostles*.[1] They put Wittgenstein in touch with the Bloomsbury younger elite (*Bloomsbury Circle*) of Cambridge University. (Garver, 2010; McGuinness, 1998) The Apostles were fascinated by him, and Bloomsbury handled him cautiously but with interest. (Griffin, 1992) Russell wrote a letter to Keynes on 11 November 1912 on Wittgenstein's election to the *Apostles*. Russell persuaded Wittgenstein to come to the first meeting to see 'how he could stand it, because from Wittgenstein's "point of view the Society is a mere waste of time". (Russell cited in Nedo, 2011, p. 15)

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[1] Apostles – was a 'secret' society, also known as *The Society of the Cambridge Conversazione Society*. It was founded in 1820 by George Tomlinson. It was not originally a secret society, but it played a very important role at the turn of the century because almost all the most influential intellectuals were members of this society. They were later so-called – the Bloomsbury Generation. For instance, David Pinsent (Wittgenstein's close friend) was considered but then rejected for membership in the Apostles. (Griffin, 1992, p. 84)

In 1912 in the letter to Russell from 22.06, Wittgenstein emphasised that logic for him was still in the 'melting-pot', however, there were some points that had become clearer for him. Wittgenstein was in the process of building his own concept of logic.

> The propositions of Logic contain ONLY APPARENT variables and whatever may turn out to be the proper explanation of apparent variables, its consequence *must* be that there are NO *logical* constants.

> Logic must turn out to be of a TOTALITY different kind than any other science. (Wittgenstein cited by McGuinness, 2001, p. 30; emphasis in the original).

In 1913 Wittgenstein finished *Notes on Logic*, where he for the first time discussed the aim of philosophy, formed his view on the distinction between philosophy and science, and criticised some of Russell's and Frege's ideas.[2] Wittgenstein wrote there that philosophy is purely descriptive, there can be no deductions in philosophy, and it is always 'over and under' but not beside natural science. "Philosophy gives no pictures of reality and can neither confirm nor confute scientific investigations. It consists of logic and metaphysics, the former its basis". (Wittgenstein, 1961, p. 93) Regarding Russell's and Frege's ideas, Wittgenstein remarked that both Frege and Russell are false in their statements "propositions are names" and "propositions correspond to complexes" accordingly. "Propositions are names of complexes" is "especially false" statement according to Wittgenstein. Expressing ideas that would be further written in *Tractatus*, Wittgenstein (1961) explained that "What corresponds in reality to a proposition depends upon whether it is true or false" (p. 93). How can we understand a proposition without knowing whether it is true or false? Wittgenstein replied, "we know what is the case if it is true and what is the case if it is false" (p. 93). Later in *Tractatus,* he wrote,

> 4.024 To understand a proposition means to know what is the case (*wissen was der Fall ist*), if it is true (*wenn er wahr ist*).

> (One can therefore understand it without knowing whether it is true or not.)
> One understands it if one understands its constituent parts. (Wittgenstein, 1922)

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[2] 'Generally, Wittgenstein's earliest investigations were in the realm of the problems with which Frege and Russell had dealt'. (Von Wright, 1955, p. 532)

Being influenced by Moore and Russell, young Wittgenstein believed that logical analysis of language would disclose the logico-metaphysical forms of facts and of their constituent objects – the substance of the world. He suggested that there is a pre-established metaphysical harmony between language and reality, i.e., the logico-syntactical forms of expressions are the forms of what they represent. In his later reflections, Wittgenstein came up with the idea that he had not solved this problem in *Tractatus*. Abstract true-false relations are not the final explanations of the process of understanding the proposition and its relationship with reality. The ordinary (living) language works in a much more complicated and intricate way than the formal logical explanation of it.

Russell and Wittgenstein, personal misunderstandings

In a letter to Russell dated 13.3.1919 from Cassino prison camp, Wittgenstein for the first time said to Russell about his new book—*Tractatus Logico-Philosophicus*.

> I've written a book called 'Logisch-Philosophische Abhandlung' containing all my work of the last six years. I believe I've solved our problems finally. This may sound arrogant, but I can't help believing it. I finished the book in August 1918 and two months after was made Prigioniere. I've got the manuscript here with me. […]

> In fact, you would not understand it without a previous explanation as it's written in quite short remarks. (This of course means that *nobody* will understand it; although I believe, it's all as clear as crystal. But it upsets all our theory of truth, of classes, of numbers and all the rest.) (Wittgenstein, 1974, p. 68)

Also, in a letter to Russell dated 19.8.1919, Wittgenstein complained Frege did not understand his ideas in *Tractatus*. "I also sent my M.S. to Frege. He wrote to me a week ago and I gather that he doesn't understand a word of it all". (p. 71) And in the other letter from 6.9.1919, he added, "I'm in correspondence with Frege. He doesn't understand a single word of my work and I'm thoroughly exhausted from giving what are purely and simply explanations". (p. 77) In a letter from 29.3.1924, Keynes wrote to Wittgenstein about *Tractatus*, "I feel certain that it is a work of extraordinary importance and genius" (p. 116). *Tractatus* was written as a reaction to the logic of Russell and Frege. Wittgenstein (1922) mentioned in the *Preface* to the *Tractatus* that "to the great works of

Frege and the writings of my friend Bertrand Russell I owe in large measure the stimulation of my thoughts" (p. 23). On the one hand, it is a true logical work (and it influenced Vienna Circle logicians and many other personalities), but on the other hand, this book is ethical. Young Wittgenstein tried to describe his version of reality, how the world is, and to go further only on particular logical problems. As von Wright wrote in Wittgenstein's biography (1955, p. 533), *Tractatus* was a synthesis of the theory of truth-functions and the idea of language as a picture of reality. McGuinness (1988) in his Introduction to '*Wittgenstein in Cambridge, Letters and Documents 1911-1951*' summed up:

> The original Abhandlung, whose completion he announced to Russell in 1915, was the product of this Cambridge period, but the additions he made to it in 1916-18 (the passages on God, freedom, and the mystical) issued rather from the next two phases in his life. Tolstoy's religion had taken hold of him in the war and the circle of young disciples of Kraus and Loos whom he met in Olmütz acted as midwives to the utterance of what he had previously and, as he thought, necessarily left unsaid. (p. 4)

In December 1919 Wittgenstein was happy to meet B. Russell at the Hague for the first time after WWI. This meeting was not very warm, the first notes of alienation and misunderstanding appeared between Russell and Wittgenstein. After their meeting, Russell in a letter to Lady Ottoline Morell wrote a wide and detailed description of the changes that had happened in Wittgenstein's mind. And this letter is very important to understand the crucial points of the turn of Wittgenstein's thought. In Russell's words, it was a turn for mysticism, he listed several thinkers who influences Wittgenstein in this way, Kierkegaard, Angelus Silesius, William James, Tolstoy, and Dostoevsky. (Russell in Wittgenstein, 1974) On his return to England, Russell wrote an introduction to Wittgenstein's *Tractatus Logico-Philosophicus*, and then helped to publish it in Germany and in English languages. In a letter from 9.4.1920, Wittgenstein wrote to Russell that he received the manuscript with his introduction. He was not in agreement with Russell "both where you're critical of me and also where you're simply trying to elucidate my point of view." However, "that doesn't matter. The future will pass judgement on us […]." Wittgenstein understood that the introduction was necessary for publication, especially from such a prominent figure as Bertrand Russell. In a previous letter from 19.1.20, he

affirmed that "the printing can't begin without it (introduction – added by me)".[3] Nevertheless, Leipzig's publisher Recalm finally did not accept his book (Wittgenstein, 1974). Wittgenstein did not like Russell's introduction translated in German at all, he wrote Russell, "All the refinement of your English style was, obviously, lost in translation and what remind was superficiality and misunderstanding". (p. 88) In 1922, Russell and Wittgenstein met each other again in Austria, Innsbruck. They argued, Wittgenstein was expressive and said unpleasant things to Russell. Since that meeting, their friendship has never been restored to what it had been before. (von Wright, 1974)

However, as years passed, Wittgenstein returned to Cambridge University on 18 January 1929. Wittgenstein being a remarkable person, had become a legendary Cambridge eccentric during his first stay at Cambridge, and he was still well-remembered. According to McGuinness (2088),

> He was to repair Russell's logic; he was to deal with Keynes's probability in two or three paragraphs; he was to show that ethics, Moore's field, did not consist of propositions at all. And perhaps this is what they wanted from him, they 'looked to him for the next big step in philosophy', as Russell told Wittgenstein's sister; and after the war they were happy (as C. D. Broad put it) 'to dance to the highly syncopated pipings of Herr Wittgenstein's flute. (p. 4)

It was necessary to obtain a Ph.D. degree to continue a research and teaching career in Cambridge. Russell offered Wittgenstein to use his work *Tractatus Logico-Philosophicus* translated into English by Ramsey eight years earlier as a thesis, finally it was accepted as his doctoral work. Frank Ramsey was formally appointed as his supervisor and B. Russell and R. Moore were his examiners. Wittgenstein passed an oral examination on June 6 and on June 18 he was awarded his PhD degree from the University of Cambridge. To be honest, this procedure was purely formal (Monk, 1990, 271). Clark in *The Life of Bertrand Russell* described the process of examination.

> Moore and Russell first chatted informally to Wittgenstein as old friends rather than as examiners and examinees. Then Russell turned to Moore. 'Go on', he said, 'you've got to ask him some questions – you're the Professor'. There was a short discussion. Russell made a brief attempt to argue that Wittgenstein was inconsistent in stating that little could be said about philosophy and that it was

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[3] Letters from 9.4.20 and 19.1.20. (Wittgenstein, 1974, p. 84)

possible to reach unassailable truth. Then the Viva ended unexpectedly with Wittgenstein clapping each of his examiners on the shoulder and exclaiming, 'Don't worry, I know you'll never understand it'. (Clark cited by Nedo, 2011, p. 8)

Earlier this month on 2 February 1929 Wittgenstein had begun work on a series of 18 manuscript volumes, he had finished his work only until 1940. On 19 June, Wittgenstein received a grant from Trinity College, arranged by Moore, Russell, and Ramsey. This one-off payment allowed him to continue his research work in Cambridge. Russell admitted in a letter to Moore from 9 March 1930,

> The Council of Trinity made a grant to Wittgenstein last June to enable him to carry on his researches on the foundations of Mathematics. There is now a question of making him a further grant; and they wish, before they decide, to have expert reports on the work he has done since the last grant was made. They have authorised me to ask you to make such a report for them. (Russell cited in Nedo 2011, p. 9)

In March and April 1930 Wittgenstein worked on the synopsis of manuscripts – later known as the Typescript 208. He discussed this text with Bertrand Russell in March, visiting him, especially for this purpose. Then Wittgenstein prepared a revision of Typescript 208 – TS 209, known as *Philosophische Bemerkungen*, and showed it again to Russel at the end of April. With this work on Russell's advice, Wittgenstein applied for a Fellowship at Trinity College in Cambridge, where he was admitted after evaluation of his work. Therefore, during the academic year 1929/1930 and Easter and summer holidays, Wittgenstein produced, Typescript 208, its revision Typescript TS 209, which were both published posthumously in 1964 in Oxford with the name *Philosophical Remarks*, and TS 210, the synopsis of the first part of volume IV. For Wittgenstein's admission to Trinity College, his work had to be evaluated by Russell and two Cambridge mathematicians J. E. Littlewood and G. H. Hardy. Russell reproduced his opinion, presented it to Trinity College Committee in his autobiography,

> The theories contained in the work of Wittgenstein are novel, very original and indubitably important. Whether they are true, I do not know. As a logician who likes simplicity, I should wish to think that they are not, but from what I have read of them I am quite sure that he should have an opportunity to work them out, since when completed they may easily prove to constitute a whole new philosophy. (Russell, 1968, vol. II, p. 200).

What Russell was really thinking about Wittgenstein, he left in his autobiography written in 1959 after Wittgenstein's death, "He was perhaps the most perfect example I have ever known of genius as traditionally conceived, passionate, profound, intense, and dominating". (Russell cited in Nedo, 2011, p. 7) In a letter to Lady Ottoline Morell in 1916, Russell admitted that Wittgenstein's critique and ideas influenced and changed his views on different philosophical problems.

> Do you remember that at the time…I wrote a lot of stuff about Theory of Knowledge, which Wittgenstein criticized with the greatest severity? His criticism… was an event of first-rate importance in my life, and affected everything I have done since. I saw he was right, and I saw that I could not hope ever again to do fundamental work in philosophy. My impulse was shattered, like a wave dashed to pieces against a breakwater. (Russell, 1968, Vol. II, p. 57)

Analysing the diaries of both Wittgenstein and Russell themselves and others, we can reconstruct the course of philosophical disputes and contradictions between them. It is interesting to trace the dynamics of changes in Russell's attitude towards Wittgenstein. In the letters from 1911, he left very different descriptions of Wittgenstein's behaviour and his character in general:

> 19 October 1911
> My German friend threatens to be an infliction, he came back with me after my lecture and argued till dinner-time – obstinate and perverse, but I think not stupid.
> 1 November 1911
> My German engineer is very argumentative and tiresome.
> 2 November 1911
> My German engineer, I think is a fool. He thinks nothing empirical is knowable – I asked him to admit that there was not a rhinoceros in the room, but he wouldn't.
> 13 November 1911
> [M]y German ex-engineer, as usual, maintained his thesis that there is nothing in the world except asserted propositions, but at last I told him it was too large a theme […]
> 16 November 1911
> My ferocious German came and argued at me after my lecture, […] He is armour-plated against all assaults of reasoning – it is really rather a waste of time talking with him.
> 29 November 1911
> I am getting to like him, he is literary, very musical, pleasant-mannered (being an Austrian) and I think really intelligent. (Russell cited by Nedo, 2011, no page)

Also, for instance, we come across interesting passages about Wittgenstein's attitude to Russell's ideas. In October 1912, Pinsent wrote about Wittgenstein's new solution to a problem in mathematical logic, "which should revolutionise lots of Symbolic Logic ... the most masterly and convincing solution too' and 'Russell…thinks…is sound". (Pinsent, 1990, p. 37) The next year in 1913 he wrote more in detail about this situation concerning symbolic logic.

> Of course [Wittgenstein] has upset a lot of Russell's work-but Russell would be the last to resent that, and really the greatness of his work suffers little thereby - as it is obvious that Wittgenstein is one of Russell's disciples and owes enormously to him. But Wittgenstein's work is really amazing - and I really believe that the mucky morass of Philosophy is at last crystallizing about a rigid theory of Logic - the only portion of Philosophy about which there is any possibility of man knowing anything Metaphysics *etc.* are hampered by total lack of data. It is like the transition from Alchemy to Chemistry. (Pinsent,1990, p. 59)

In a letter sent by Ramsey to his mother in September 1923, after a meeting with Wittgenstein, he wrote that Wittgenstein was a 'little annoyed' that Russell was doing a new edition of *Principia Mathematica* because "he thought he had shown R [Russell] that it was so wrong that a new edition would be futile. It must be done altogether afresh" (Ramsey in Nedo 2011, p. 21). Russell, on the other hand, did not understand Wittgenstein's *Tractatus*. Janik (2001) assumed that the real reason for this misunderstanding was in absolutely different approaches. Russell, at that time, advocated logical positivism, while Wittgenstein's early ideas were based on a "Hertzian perspective on philosophy." In 1930 Wittgenstein said to his friend Maurice O'C Drury (Drury, 1984) his opinion on Russell's books, they "should be bound in two colours, those dealing with mathematical logic in red – and all students of philosophy should read them; those dealing with ethics and politics in blue – and no one should be allowed to read them" (p. 112). In the notes from 1937, published in *CV*, Wittgenstein remembered how during their conversations with Russell, the latter often exclaimed "Logic's hell!" Here he explained the nature of the difficulties when they both tried to resolve logical problems which were inextricably connected with the peculiarities of the language. According to Wittgenstein, this phrase in the best way expresses the feeling that they had when they were thinking about the problems of logic. It expresses the "immense difficulty" of the problems of logic, "their hard and slippery texture". (Wittgenstein,1980, p. 30[e])

I believe our main reason for feeling like this was the following fact, that every time some new linguistic phenomenon occurred to us, it could retrospectively show that our previous explanation was unworkable. (We felt that language could always make new, and impossible, demands; and that this made all explanation futile.) (p. 30ᵉ)

Wittgenstein blamed Russell for the lack of depth and insufficient attention to the so-called ordinary life and its problems. In *Zettel* Wittgenstein (1967) wrote,

Some philosophers (or whatever you like to call them) suffer from what may be called "loss of problems". Then everything seems quite simple to them, no deep problems seem to exist any more, the world becomes broad and flat and loses all depth, and what they write becomes immeasurably shallow and trivial. Russell and H. G. Wells suffer from this. (p. 82ᵉ)

In the lectures recorded by Moore (Wittgenstein & Moore, 2016) we meet: "Russell's view of structure of propositions was far too simple. Grammar of different kinds of words is much more different than you would expect". (p. 253)

Wittgenstein's critique of Bertrand Russell's basic ideas

Russell in the first decade of the 20th century tried to develop Gottlob Frege's logical and mathematical ideas, among them, *predicate calculus*. He sought to realize the leading idea of Frege's research—to reduce all mathematics to logic and, thus, to build a reliable and obvious foundation for it, as well as to answer philosophical questions about the nature of mathematics and the source of the irrefutability of its truths.

Russell had already been able to recognize the difficulties and problems that Frege's approach ran into. Russell formulated the paradox that shook the system of logical foundations of mathematics built by Frege. Russell's paradox, like several other paradoxes, is based on the fact that an expression is applied to itself (or, equivalently, a certain property is asked whether it characterises itself). When an expression is applied to itself, it is said to be used self-referentially. Self-referentiality is fraught with paradoxes (e.g., the 'Liar paradox').[4] Another significant difficulty that Frege's approach (Frege

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[4] Liar paradox is the statement of a liar, that he is lying, 'I am lying'. If the liar is lying, then the liar is telling the truth. But if the liar is telling the truth, it means that he lied. Here we face a contradiction trying to assign

1892a, pp. 41-42; 1893, pp. 19-20) encountered (Beaney 1997, pp. 384-5) was the problem of "empty names" (or definite description), that is, names that make sense but have no meaning (they do not refer to anything). The difficulty was in fact that no logical criteria in Frege's system could distinguish empty names from names that have meaning, for this was a matter of information. This circumstance created great inconvenience in the implementation of logical conclusions. Realising this difficulty, Frege put forward the requirement that there should be no empty names in a logically correct language - they should have been eliminated in advance. Russell (1905) remarked that Frege's approach was "plainly artificial".

Russell, on the other hand, took a different path, resulting in a completely different concept of language meaning expressions. Russell was not only a mathematician but also a philosopher, so the context of his thoughts and the range of problems he discussed were much wider than that of Frege. As a young man, Russell was influenced by the philosophy of British neo-Hegelianism, or absolute idealism. (See Griffin, 1992; Baker & Hacker, 1980a, 1980b) According to absolute idealism, the world appears as an indivisible and unified whole. No fact or element is self-sufficient. Therefore, any attempt to consider the elements of the whole in isolation leads to distortion and delusion. Only a complete and holistic consideration can claim the truth. All reasoning about the absolute must be *a priori* because experience is limited to serve as a basis for it. Relationships within the whole were seen as internal and necessary. Here, external, and accidental relations were not recognized.

Studying Leibniz (see Griffin, 1992; Russell, 1900), Russell became aware of the importance of relations, in particular ordering relations were essential for mathematics. Neo-Hegelians rejected relations or insisted that any relations are internal. Russell understood that his analysis of mathematics is impossible without relations, and he broke with neo-Hegelianism. Instead, he had adopted a position that could be called "absolute realism", the type of ontology that includes everything that could be thought of or

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to this statement a classical binary truth-value. If 'this sentence is false' is True, then it is False. However, the statement says that it is false, and if it is False, it must be True.

mentioned. "The logic which I shall advocate is atomistic, as opposed to the monistic logic of the people who more or less follow Hegel". (Russell, 1918, p. 495).

In 1900, Russell discovered Peano's method of symbolic logic. This year he finished the draft of his *The Principles of Mathematics.* (Russell, 1903) Continuing analysis of principles of mathematics, in 1901 Russell came up with his famous *class paradox* (also known as a Russell-Zermelo[5] paradox. He discovered that a class of all classes which are not members of themselves both was and was not a member of itself (according to set-theoretical principles current at his time), or, in other interpretations, the set of all sets that are not members of themselves will be a member of itself if and only if it is not a member of itself.

> The axiom that all referents with respect to a given relation form a class seems, however, to require some limitation, and that for the following reason. We saw that some predicates can be predicated of themselves. Consider now those … of which this is not the case. … [T]here is no predicate which attaches to all of them and to no other terms. For this predicate will either be predicable or not predicable of itself. If it is predicable of itself, it is one of those referents by relation to which it was defined, and therefore, in virtue of their definition, it is not predicable of itself. Conversely, if it is not predicable of itself, then again it is one of the said referents, of all of which (by hypothesis) it is predicable, and therefore again it is predicable of itself. This is a contradiction. (Russell, 1986, Vol. 3, p. 195)

This significant paradox was unresolved in classical logic, all sentences were entailed by a contradiction. Propositional functions – "'x is a set' cannot be applied to themselves because self-application leads to a vicious circle". (Russell, 1963, Vol. I., p. 67; see Poincaré, 1905, p. 307) Trying to solve this problem Russell developed his *theory of types* during 1903-1908 (arranging all sentences into a hierarchy of levels). (Russell, 1908)

First, in 1903, he introduced the 'simple' version of this theory, and in 1908 the 'ramified theory'. He was concerned about this theory because he realised that there could not be a single solution. Together with Whitehead, Russell suggested applying the *axiom of reducibility* to resolve some most worrisome aspects of their theory of types.

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[5] Zermelo noticed the similar contradiction in about 1897-1902, anticipating Russell. See Ebbinghaus & Peckhaus (2007, pp. 43-38).

However, critics claimed that this axiom was rather simple (Linsky, 1999, 2002; Wahl, 2011). Also, during this period Russell defended *logicism*, a theory according to which mathematics is reducible to logic. Russell suggested two main theses in his programme of logicism, (i) all mathematical truths can be translated into logical truths; (ii) all mathematical proofs can be recast as logical proofs. "The fact that all Mathematics is Symbolic Logic is one of the greatest discoveries of our age; and when this fact has been established, the remainder of the principles of mathematics consists in the analysis of Symbolic Logic itself". (Russell, 1903, p. 5) Russell tried to use logic to clarify not only the issues of mathematics but also to clarify issues of philosophy.

Frank Ramsey, a mathematician and Wittgenstein's friend, criticised Russell's *Theory of Types*. In a letter to Wittgenstein dated February 20, 1924, he wrote that he "made out the proper solution rather in detail of some of the contradictions which made Russell's Theory of Types unnecessarily complicated." For Ramsey this complicity "made him [Russell] put in the Axiom of Reducibility." Ramsey was reading the manuscript of the new material Russell was putting into the *Principia Mathematica*. And he agreed with Wittgenstein, that "it is of no importance." All this new stuff "is a clever proof of mathematical induction without using the axiom of reducibility." And there were neither new ideas nor fundamental changes. Russell accepted only one of Wittgenstein's critical remarks on "that it is nonsense to put an adjective where a substantive ought to be which helps in his theory of types." Moreover, Russell "indignantly denied ever having said that vagueness is a characteristic of the physical world". (Nedo, 2011, p. 20) In a letter to Moore dated 6.2.1924 Ramsey described in detail his critique to Russell's *Theory of Types*, agreed with Wittgenstein, and mentioned his new project.

> I am working based on Wittgenstein's work, which seems to me to show that Principia is wrong not merely in detail but fundamentally. I have got Russell's manuscript of the stuff he is inserting into the new edition and it seems to me to take no account of Wittgenstein's work at all. There is a new Theory of Types without the axiom of reducibility, on which however Russell hasn't succeeded improving a lot of ordinary mathematics, whose truth, he concludes, remains doubtful. But I have got on Wittgenstein's principles a new theory of types without any doubtful axioms, which gives all the results of Russell's one and solves all the contradictions.

But Wittgenstein and I think it wrong to suppose with Russell that mathematics is more complicated formal logic (tautologies) and I am trying to make definite the vague idea Wittgenstein has of what it does consist of. If I am successful, I think it will illuminate not only mathematics but physics also because a successful theory of mathematics will help one to separate and give a true account of the a priori element in physics. (This certainly exists for 'this is not both red and blue' is a priori). (McGuinness, 2006, p. 21)

Wittgenstein in *Zettel* (1970) explained that the practical purpose of Russell's *Theory of Types* is, "Russell makes us realize that we must sometimes put restrictions on the expression of generality in order to avoid having consequences drawn from it" (p. 120ᵉ). In the notes of Wittgenstein's (Wittgenstein & Moore, 2016) lectures recorded by Moore we can find the following: "This explains what's wrong with Russell's theory of types: a theory of types is a grammar, and must not mention meanings of words". (p. 30) Since about the end of 1929, Wittgenstein had been developing his new 'later' approach to language, introducing consequently such terms as "language-games", "family resemblance", "grammar", "rules", "forms of life", making a transition from the *logical* point of view on language to *anthropological.* Hence, in 1932 he denied the idea that mathematics and logic are one building with logic as the foundation, that mathematics derives from logic, and "Russell's calculus is one calculus among the others. It is a bit of mathematics". (Wittgenstein, 2001, p. 13) In *TBT* Wittgenstein (2005) explained: "Through Russell, but especially through Whitehead, there entered into philosophy a false exactitude that is the worst enemy of real exactitude. At the bottom of this lies the erroneous belief that a calculus could be the metamathematical foundation of mathematics". (p. 376ᵉ)

Logical atomism

In the first lecture of his lecture course[16] entitled *The Philosophy of Logical Atomism* (1918-1919), Russell (1918) wrote:

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[6] "The following articles are the lectures of a course of eight lectures delivered in London in the first months of 1918, and are very largely concerned with explaining certain ideas which I learnt from my friend and former pupil Ludwig Wittgenstein. I have had no opportunity of knowing his views since August, 1914, and I do not even know whether he is alive or dead. He has therefore no responsibility for what is said in these lectures beyond that of having originally supplied many of the theories contained in them.' (Russell, 1928, p. 495).

> The kind of philosophy that I wish to advocate, which I call Logical Atomism, is one which has forced itself upon me in the course of thinking about the philosophy of mathematics, although I should find it hard to say exactly how far there is a definite logical connection between the two. (p. 495)

"When I say that my logic is atomistic," Russell continued, "I mean that I share the common-sense belief that there are many separate things; I do not regard the apparent multiplicity of the world as consisting merely in phases and unreal divisions of a single indivisible Reality". (p. 496) The concept is called logical atomism because the 'atoms', according to Russell, that he wished "to arrive at as the sort of last residue in analysis are logical atoms and not physical atoms." (p. 496) This is the last given to which logical analysis arrives, i.e., the elements that are no longer decomposable by logical analysis. From Russell's point of view, 'physical objects,' 'physical events,' 'matter,' 'space,' 'consciousness,' and 'subject' are all logical constructions built from '*logical atoms*,' and logical connectives according to the rules of logic. Therefore, they are decomposable by logical analysis. What are '*logical atoms*?' As we can see, Russell defines them by pointing to their place in the procedure of logical analysis. How is a logical analysis conducted? Russell defines it as a transition from something unclear, indefinite, or imprecise (vagueness and inaccuracy are not commonly realised due to the habitual expression) to clear precise and definite concepts. Russell considers as clear, precise, and definite the concepts whose connection with the signified is clear and understandable, so that there can be no misunderstanding about their meaning and whether what they signify exists.

Russell proposes to proceed as a self-evident assumption that there are facts in the world and that we form judgements about these facts and express them in sentences. A fact is something that can be expressed by a sentence. It can also be said that a fact is what makes a sentence true or false. The facts themselves are neither true nor false. Logic distinguishes between private ('Some flowers are red') and general ('All men are mortal') sentences, as well as positive ('Man is a rational animal') and negative ('Socrates is not Plato's father'). Accordingly, Russell also classifies facts, they also turn out to be particular and general, positive, and negative. Ergo, Russell describes reality through the prism of logical concepts so that reality reflects the accepted method of logical analysis.

The grammatical category of sentences is distinguished by the property that they can be true or false. Thus, Russell argues, they cannot be considered as names, as Frege did. Sentences are not names of facts, for if we recognized true sentences as names of facts, then what would false sentences be called?

> It is very important to realise such things, for instance, as that propositions are not names for facts. It is quite obvious as soon as it is pointed out to you, but as a matter of fact I never had realised it until it was pointed out to me by a former pupil of mine, Wittgenstein. It is perfectly evident as soon as you think of it, that a proposition is not a name for a fact, from the mere circumstance that there are two propositions corresponding to each fact. (Russell, 1918, p. 507)

Russell assigns the central position to the category of the sentence and raises the question of its understanding. A sentence is a complex symbol made up of other, simpler symbols.

> A proposition is just a symbol. It is a complex symbol in the sense that it has parts which are also symbols, a symbol may be defined as complex when it has parts that are symbols. In a sentence containing several words, the several words are each symbol, and the sentence composing them is therefore a complex symbol in that sense. (Russell, 1918, p. 507)

A sentence is understandable only if the simple symbols that form it are understandable. In general, as Russell notes, symbols must be studied so as not to confuse the properties of symbols and the properties of the things they denote. Because of this confusion, all sorts of philosophical problems associated with 'existence' arise. Do numbers, properties exist? Is there a set of all sets? A symbol, Russell explains, is all that matters. Different types of symbols relate differently to what they signify. The value of a complex symbol is determined by the values of its constituent simple ones (a symbol is simple if its parts are not themselves a symbol).

> There are different kinds of symbols, different kinds of relation between symbol and what is symbolized, and very important fallacies arise from not realizing this. The sort of contradictions about which I shall be speaking in connection with types in a later lecture all arise from mistakes in symbolism, from putting one sort of symbol in the place where another sort of symbol ought to be. (Russell, 1918, p. 508)

Speaking about symbolism, Russell explained,

> I am using it in a sense to include all language of every sort and kind, so that every word is a symbol, and every sentence, and so forth. When I speak of a symbol, I simply mean something that 'means' something else, and as to what I mean by 'meaning' I am not prepared to tell you. (p. 508)

What is the meaning of a simple symbol? How can one understand it?

> I think that the notion of meaning is always more or less psychological, and that it is not possible to get a pure logical theory of meaning, nor therefore of symbolism. I think that it is of the very essence of the explanation of what you mean by a symbol to take account of such things as knowing, of cognitive relations, and probably also of association. The theory of symbolism and the use of symbolism is not a thing that can be explained in pure logic without taking account of the various cognitive relations that you may have to things. (p. 508)

Russell answers that we can understand the meaning only through direct acquaintance with the object denoted by this symbol. By *direct acquaintance,* he means the presence of a given object in the subject's sensory experience of object symbols, each of which corresponds to a component of the complex. Russell gave an illustration of what he meant,

> For instance, the word 'Socrates,' you will say, means a certain man; the word 'mortal' means a certain quality; and the sentence 'Socrates is mortal' means a certain fact. But these three sorts of meaning are entirely distinct, and you will get into the most hopeless contradictions if you think the word 'meaning' has the same meaning in each of these three cases. It is very important not to suppose that there is just one thing which is meant by 'meaning,' and that therefore there is just one sort of relation of the symbol to what is symbolized. A name would be a proper symbol to use for a person; a sentence (or a proposition) is the proper symbol for a fact. (Russell, 1918, p. 509)

The concept of logical atomism is both logical and metaphysical at the same time. As a logician, Russell considers the structure of an ideal language; as a metaphysician, he claims that reality has the same structure. In the past metaphysical doctrines had often based their general idea of the structure of reality on some scientific discipline. Thus, the Pythagoreans asserted that reality has a mathematical essence, and used the language of mathematics to describe it. In the 18th-19th century, Newtonian mechanics was applied as a model for describing and explaining reality. Nowadays, evolutionary biology, systems theory, or computer science often become the basis of philosophical reasoning about the general nature of reality. For Russell, as we can see, logic played this role. He believed

that the largest metaphysical systems, e.g., the systems of Spinoza, Leibniz, Hegel, and Bradley, are based precisely on the logical views of their authors and essentially depend on the fact that in traditional logic the sentence was assigned a subject-predicate structure. Corresponding to such logic, metaphysics describes the world as a single substance and its attributes. Therefore, Russell consciously undertook the construction of new metaphysics corresponding to the new logic developed by G. Frege, G. Peano, A. Whitehead, and himself.

One of the main features of this metaphysics is the assumption of various and independent 'simple objects' (Russell calls them particulars). What are these 'simple objects?' In what sense are they simple? These objects are independent, which means that there is no causal relationship between them. These are not the objects of physics. Physical objects and physical laws, according to Russell, are logical constructions. They can be analysed. The analysis ends when it comes to simple symbols and, accordingly, to simple objects. The latter can only be named, they cannot be described. I suppose Russell was referring to 'sense data.' Here he followed the tradition of British empiricism. This is also indicated by Russell's distinction between two kinds of knowledge, that can help to obtain knowledge about simple objects and about complexes - knowledge by direct acquaintance and knowledge by description. The method of logical analysis itself was specific to Russell's concept of logical atomism. It is based on the fact that the naming expression is replaced by a description. With this, we get rid of the naming expression and from the problem of the existence of what is denoted by this expression. This is Russell's approach to many problems in the philosophy of mathematics. Instead of arguing whether there are (exist), and if they do exist, therefore in what sense, numbers, sets, and other mathematical objects; Russell constructs definitions that replace them—descriptions of known properties and relationships. Then, in all sentences in which there are expressions for numbers and sets, they are replaced by the corresponding descriptions. In metaphysical disputes, Russell says, phrases such as 'God exists,' 'Subject exists,' and so on are common. Problems of existence are typical for metaphysical disputes. There is a tendency to declare everything that is denoted by some noun, numbers, sets, properties, relations, Pegasus, the 'Golden Mountain', a round

square, etc., as existing. To avoid this, one should subject phrases that assert existence or the non-existence of something, to logical analysis. If we consider the sentence "Pegasus never existed," we understand its meaning. For the understanding of a sentence, an understanding of the simple names included in it is required, but we cannot understand the meaning of the name 'Pegasus' precisely because Pegasus never existed, and we do not know what it is. The solution to this difficulty lies in the fact that "Pegasus" is not a constituent of this proposal, as the analysis should show. The fact is that "Pegasus" is not a name, but a description - a propositional function, "Horse (*x*) and Winged (*x*)." Therefore, the sentence "Pegasus never existed," as a result of Russell's analysis, becomes the sentence, "There has never been an *x* such that the Horse (*x*) and the Winged (*x*) and x is the only object that is both the Horse and the Winged." Here, as we see, the word "Pegasus" is missed, and therefore there is no reason to raise the question of the existence of Pegasus, real or ideal. But why does Russell believe that such an analysis is legitimate? Because, as he explains, the sentence must consist of constituents that uniquely correspond to the constituents of the corresponding fact. However, in many sentences, some parts do not correlate with the constituents of the fact. Thus, in the phrase "Walter Scott is the author of Waverley," the phrase "the author of Waverley" is not a constituent of the sentence, because in real facts there is no corresponding constituent. There is only one real person - Walter Scott. And the combination 'the author of Waverley' is a description, and it should be eliminated so as not to create the impression that it is a matter of establishing the existence of two persons where there is only one person. Russell considered his method of eliminating descriptions to be an effective tool for solving a wide variety of philosophical problems, for instance, for criticising philosophical theories that abuse the concept of the absolute or invent various 'kinds' and 'modes' of existence.

Critique of Russell's 'logical atomism' by early Wittgenstein

The main statements:

First, Wittgenstein was not satisfied with Russell's rationale for the procedure of logical analysis, which connects logical analysis with the empiricist theory of knowledge.

Wittgenstein is concerned with showing the justification and necessity of logical analysis, but he tries to achieve this in quite different ways.

Second, it is important for Wittgenstein to grasp the true meaning of the analytic method. It is not enough for him that this method allows one to reveal and eliminate the confusion that existed earlier in someone's philosophical concepts, i.e., the eternal questions of the meaning of life, of good and evil, of death and immortality. He felt the need to relate the method developed by him and Russell to these questions and understand its significance and its possibilities against their background.

Third, Wittgenstein considered it necessary to make a much more rigid and consistent separation of logical and factual reasoning than Russell did. It is obvious to him that those logical truths cannot be discovered by examining some existing circumstances. The question of the nature and status of logic must be defined more correctly than was the case in Russell's reasoning.

Fourth, Wittgenstein in his *Tractatus* resolved several logical difficulties and special questions concerning the foundations of mathematics.

Critique of Russell's 'logical atomism' by middle and later Wittgenstein

In his lectures on philosophy from 1932-33, Wittgenstein (2001) was talking that Russell and he expected to find the first elements (or individuals), and thus the possible atomic propositions by logical analysis. Russell thought that the result of the final analysis would be the subject-predicate proposition (2-term relations). But this was wrong, according to Wittgenstein, Russell took logical analysis as it was like the chemical (scientific) analysis. And the fault of both Russell and Wittgenstein was that they did not give any examples of atomic propositions or of individuals. They both in different ways pushed the question of examples aside. Also, atomic propositions should not contain 'and', 'or', etc.

> Russell and I both expected to find the first elements, or "individuals", and thus the possible atomic propositions, by logical analysis. Russell thought that subject-predicate propositions, and 2-term relations, for example, would be result of a final analysis. This exhibits a wrong idea of logical analysis: logical analysis is taken as being like chemical analysis. And we were at fault for giving no examples of atomic

propositions or of individuals. We both in different ways pushed the question of examples aside. […]

Atomic propositions are not the result of an analysis which has yet to be made. We can talk of atomic propositions if we mean those which on their face do not contain "and", "or", etc., or those which in accordance with methods of analysis laid down do not contain these. There are no hidden atomic propositions. (Wittgenstein, 2001, p. 11)

Wittgenstein continued that with the means of the concept of language-games, he wanted to show the vague cases in which we use such words as 'language', 'proposition', 'sentence'. "There are many things, such as orders, which may or may not be call propositions; and not only one game can be called language." Moreover, "Language-games are a clue to the understanding of logic". (p. 12) Wittgenstein said, "what we call logic plays a different role from that which Russell and Frege supposed". (p. 13) That means:

When Frege tried to develop mathematics from logic he thought that calculus of logic was *the* calculus, so that what followed from it would be correct mathematics. Mathematics and logic were one building, with logic the foundation. This I deny; Russell's calculus is one calculus among others. (p. 13).

Moreover, Wittgenstein (1970) criticised both Russell and Frege arguing that "they take concepts as, as it were, properties of things. But it is very unnatural to take the words man, tree, treatise, circle, as properties of a substrate". (p. 122[e]) Wittgenstein criticised his contemporaries for the *misleading* use of *calculus* trying to apply it to the problems of language and logic. "A calculus can't be true or false; nor more or less fundamental (as Russell and Frege believed of logical calculus: it is merely a bit of mathematics). Tarski has added a new calculus; & made a mistake as to how it can be used". (Wittgenstein & Moore, 2016, p. 31) For instance, Martin Heidegger, the other prominent philosopher of language of the 20th century, agreed that it is erroneous to consider language as calculus, moreover as well as later Wittgenstein, he criticised the approach of logicians to find the foundation of language in logic. Furthermore, both later Wittgenstein and Heidegger realised that this artificial placing of language in the frames of logic leads to tautologies. According to Heidegger (2001), "The understanding that is schooled in logic, thinking of everything in terms of calculation and hence usually overbearing, calls this proposition an empty tautology". (p. 188) To conclude, for Wittgenstein (2005) *calculating* is an *activity*

that differs from all the others, "especially as concerns its reason" (p. 181e). The calculation process consists of acts, each of which is an individual act of thought. In language, we meet another state of affairs, as well as the other relationship between language and thought. In general, Wittgenstein (2005) says about his later work: "Our task is not to improve our language, to make it more exact, or possibly even to try to replace it with an 'ideally exact' one. We have absolutely no concept of such a language". (p. 200^e)

Conclusion: Basic statements of Wittgenstein's critique of Russell's ideas

To conclude, I will list the fundamental statements of Wittgenstein's critique of Russell and Frege as well as his own earlier ideas.

1) "Impression that logic is not a thing within our control"; (Wittgenstein, 2001, p. 99)

2) "Logicians make up an ideal language to which our normal language only approximate"; (p. 99)

3) "Logic describes the use of language in a vacuum"; (p. 99)

4) "Logical Concepts are Pseudo-concepts"; (Wittgenstein, 2016, p. 24)

5) "Logicians have had an obsession that the life of a proposition is the copula 'is'", however, not every proposition can be reduced to subject-copula-adjective form (subject-predicate); (Wittgenstein, 2001, p. 100)

6) "We have created an enormously complicated language for our use", it cannot be reduced to simplifications, otherwise, we get errors and erroneous understanding of the whole phenomenon of language; (p. 101)

7) A language in general is a conglomerate including special languages (the languages of chemistry, meteorology, physics, etc.) consisting of a "mixture of descriptions, hypotheses, questions, orders, etc., but any list we made of this would be entirely inadequate"; (p. 102)

8) "Russell & Frege translated ordinary language into a symbolism"; (Wittgenstein, 2016, p. 222; emphasis in the original)

9) "96. Russell and Frege construe a concept as a sort of property of a thing. But it is very unnatural to construe the words 'man', 'tree, treatise', 'circle' as properties of a substratum". (Wittgenstein, 1998, p.120)

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